

considered as demonstrated that, for most ordinary cases of large bridges, calculations founded on equation (26) may be confidently relied on. It need scarcely be remarked that these are much more simple and easy than those founded on the more exact but complex equations above given.

In smaller bridges, however, the error of the approximate process will be more considerable, and the process above given may be applied with advantage to its correction.

In concluding this paper, the author desires to record his thanks to his young friend, Mr. Henry Reilly, for the patience and skill with which he made, in detail, all the intricate calculations of the numerical values of the various functions involved in the above demonstration.

“Remarks on Mr. Heppel's Theory of Continuous Beams.” By W. J. MACQUORN RANKINE, C.E., LL.D., F.R.S. Received December 22, 1869*.

1. *Condensed form of stating the Theory.*—The advantages possessed by Mr. Heppel's method of treating the mathematical problem of the state of stress in a continuous beam will probably cause it to be used both in practice and in scientific study.

The manner in which the theory is set forth in Mr. Heppel's paper is remarkably clear and satisfactory, especially as the several steps of the algebraical investigation correspond closely with the steps of the arithmetical calculations which will have to be performed in applying the method to practice.

Still it appears to me that, for the scientific study of the principles of the method, and for the instruction of students in engineering science, it may be desirable to have those principles expressed in a condensed form; and with that view I have drawn up the following statement of them, which is virtually not a new investigation, but Mr. Heppel's investigation abridged.

Let $(x=0, y=0)$ and $(x=l, y=0)$ be the coordinates of two adjacent points of support of a continuous beam, x being horizontal. Let y and the vertical forces be positive downwards.

At a given point x in the span between those points let μ be the load per unit of span, and EI the stiffness of the cross-section, each of which functions may be uniform or variable, continuous or discontinuous.

In each of the following double and quadruple definite integrals, let the lower limits be $x=0$.

$$\left. \begin{aligned} \iint \mu dx^2 = m; \quad \iint \frac{dx^2}{EI} = n; \\ \iint \frac{xdx^2}{EI} = q; \quad \iint \frac{dx^2}{EI} \iint \mu dx^2 = F. \end{aligned} \right\} \dots \dots (1)$$

When the integrations extend over the whole span l , that will be denoted by affixing 1; for example, m_1, n_1 , &c.

* Read January 27, 1870. See vol. xviii. p. 178.

Let $-P$ be the upward shearing-force exerted close to the point of support ($x=0$), Φ_0 the bending moment, and T the tangent of the inclination, positive downwards, at the same point. Then, by the general theory of deflection, we have, at any point x of the span l , the following equations:—

$$\text{moment,} \quad \Phi = \Phi_0 - Px + m; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\text{deflection,} \quad y = Tx - Pq + \Phi_0 n + F. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Let Φ_1 be the moment at the further end of the span l , and suppose it given. This gives the following values for the shearing-force P and slope T at the point ($x=0$):—

$$P = \frac{\Phi_0 - \Phi_1 + m_1}{l}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and because $y_1=0$,

$$T = \frac{Pq_1 - \Phi_0 n_1 - F_1}{l} = \Phi_0 \left(\frac{q_1}{l^2} - \frac{n_1}{l} \right) - \frac{\Phi_1 q_1}{l^2} + \frac{m_1 q_1}{l^2} - \frac{F_1}{l}. \quad . \quad . \quad (5)$$

Consider, now, an adjacent span extending from the point of support ($x=0$) to a distance ($-x=l'$) in the opposite direction, and let the definite integrals expressed by the formulæ (1), with their lower limits still at the same point ($x=0$), be taken for this new span, being distinguished by the suffix -1 instead of 1. Let $-T'$ be the slope at the point of support ($x=0$). Then we have for the value of that slope,

$$-T' = \Phi_0 \left(\frac{q_{-1}}{l'^2} - \frac{n_{-1}}{l'} \right) - \frac{\Phi_{-1} q_{-1}}{l'^2} + \frac{m_{-1} q_{-1}}{l'^2} - \frac{F_{-1}}{l'}. \quad . \quad . \quad (5A)$$

Add together the equations (5) and (5A), and let $t = T - T'$ denote the tangent of the small angle made by the neutral layers of the two spans with each other in order to give imperfect continuity. Then, after clearing fractions, we have the following equation, which expresses *the theorem of the three moments* in Mr. Heppel's theory:—

$$0 = \Phi_0 \left\{ q_1 l'^2 + q_{-1} l^2 - n_1 l l'^2 - n_{-1} l' l^2 \right\} - \Phi_1 q_1 l'^2 - \Phi_{-1} q_{-1} l^2 \left\{ \begin{array}{l} + m_1 q_1 l'^2 + m_{-1} q_{-1} l^2 - F_1 l l'^2 - F_{-1} l' l^2 - t l^2 l'^2. \end{array} \right\} \quad . \quad (6)$$

That equation gives a linear relation between the bending moments Φ_{-1} , Φ_0 , Φ_1 at any three consecutive points of support, and certain known functions of known quantities. In a continuous girder of N spans there are $N-1$ such equations and $N-1$ unknown moments; for the moments at the end most supports are each $=0$. The moments at the intermediate points of support are to be found by elimination; which having been done, the remaining quantities required may be computed for any particular span as follows:—The inclination T at a point of support by equation (5); the shearing force P at the same point by equation (4); the deflection y and moment Φ at any point in that span by equations (3) and (2). Points of maximum and minimum bending moment are of course found by making $\frac{d\Phi}{dx} = 0$; and points of inflection by making $\Phi = 0$.

2. *Case of a uniform girder with an indefinite number of equal spans, uniformly loaded; loads alternately light and heavy.*—The supposition just described forms the basis of the formulæ given in a treatise called ‘A Manual of Civil Engineering,’ page 288; and it therefore seems to me desirable to test those formulæ by means of Mr. Heppel’s method.

The cross-section of the whole girder and the load on a given span being uniform, the definite integrals of the formulæ (1) take the following values :—

$$m = \frac{\mu x^2}{2}; \quad n = \frac{x^2}{2EI}; \quad q = \frac{x^3}{6EI}; \quad F = \frac{\mu x^4}{24EI} = \frac{mq}{2x} \dots (7)$$

The values of those integrals for the complete span are expressed by making $x=l$.

The values of n and q are the same for every span. In the values of m and F , the load μ per unit of span has a greater and a less value alternately. Let w_0 be the weight per unit of span of the girder, with its fixed load, w_1 that of the travelling load (increased, if necessary, to allow for the additional straining effect of motion); then the alternate values of μ are

$$\mu = w_0; \quad \mu' = w_0 + w_1 \dots (8)$$

The moments at the points of support are all equal; that is, $\Phi_0 = \Phi_1 = \Phi_{-1}$.

Equation (6) now becomes the following (the common factor l^3 having been cancelled) :—

$$0 = -2\Phi_0 n_1 + F_1 + F_{-1} - tl;$$

giving for the bending moment at each point of support

$$\Phi_0 = \frac{F_1 + F_{-1} - tl}{2n_1} = \frac{2w_0 + w_1}{24} \cdot l^2 - \frac{t}{l} EI \dots (9)$$

If t be made $=0$, so that the continuity is perfect, this equation exactly agrees with the formula at page 289 of the treatise just referred to; and the same is the case with the following formulæ for the shearing-forces and slopes close to a point of support, and for the moments and deflections at other points :—

$$\left. \begin{array}{ll} \text{Shearing-force, light load,} & P = \frac{w_0 l}{2}; \\ \text{Shearing-force, heavy load,} & P_1 = \frac{w_0 + w_1}{2} \cdot l. \end{array} \right\} \dots (10)$$

$$\left. \begin{array}{ll} \text{Slope, light load,} & T = \frac{Pq_1 - \Phi_0 n_1 - F_1}{l} = \frac{t}{2} - \frac{w_1 l^2}{48EI}; \\ \text{Slope, heavy load,} & -T' = \frac{t}{2} + \frac{w_1 l^2}{48EI}. \end{array} \right\} \dots (11)$$

Moment, light load,

$$\Phi = \Phi_0 - Px + m = -\frac{t}{l}EI + \frac{2w_0 + w_1 l^2}{24} - \frac{w_0 l x}{2} + \frac{w_0 x^2}{2}; \quad \left. \vphantom{\frac{t}{l}EI} \right\} \quad (12)$$

Moment, heavy load,

$$\Phi' = -\frac{t}{l}EI + \frac{2w_0 + w_1 l^2}{24} - \frac{w_0 + w_1 l x}{2} + \frac{w_0 + w_1}{2} \cdot x^2.$$

$$\left. \begin{array}{l} \text{Central moment, light load, } \Phi\left(x = \frac{l}{2}\right) = -\frac{t}{l}EI + \frac{w_1 - w_0 l^2}{24}; \\ \text{Central moment, heavy load, } \Phi\left(x = \frac{l}{2}\right) = -\frac{t}{l}EI - \frac{w_0 + 2w_1 l^2}{24}. \end{array} \right\} \quad (13)$$

Central deflection, light load,

$$y = Tx - Pq + \Phi_0 n + F\left(\text{with } x = \frac{l}{2}\right) = \frac{tl}{8} + \frac{w_0 - 2w_1}{384EI} \cdot l^4;$$

Central deflection, heavy load,

$$y = -T'x - P'q + \Phi_0 n + F\left(\text{with } x = -\frac{l}{2}\right) = \frac{tl}{8} + \frac{w_0 + 3w_1}{384EI} \cdot l^4. \quad \left. \vphantom{\frac{tl}{8}} \right\} \quad (14)$$

COMMUNICATIONS RECEIVED SINCE THE END OF THE SESSION.

“Researches into the Chemical Constitution of the Opium Bases.
—Part IV. On the Action of Chloride of Zinc on Codeia.” By
AUGUSTUS MATTHIESSEN, F.R.S., Lecturer on Chemistry at St.
Bartholomew’s Hospital, and W. BURNSIDE, of Christ’s Hos-
pital. Received June 23, 1870.

On endeavouring to prepare apomorphia by a cheap method, Mr. Mayer and one of us heated morphia with chloride of zinc, to see whether the elements of water could not be abstracted by this reagent (the results of this reaction have not yet been published). Apomorphia having been obtained in this manner, it seemed possible that apocodeia, that is codeia minus the elements of water, might be prepared by a similar reaction. On trying the experiment a new base was obtained, which proved on analysis to be apocodeia.

When hydrochlorate of codeia is heated with an excess of a concentrated solution of chloride of zinc, to a temperature varying between 170° and 180° C., for about 15 minutes, the decomposition takes place; on cooling a yellowish-brown tarry mass separates from the liquid, which on further cooling may be drawn into thin threads, and thus obtained almost free from the excess of the chloride of zinc. This amorphous silk-like mass is almost pure hydrochlorate of apocodeia. To obtain the base in a pure state from this substance, the following method was employed:—

The hydrochlorate was dissolved in hot water and precipitated by hy-